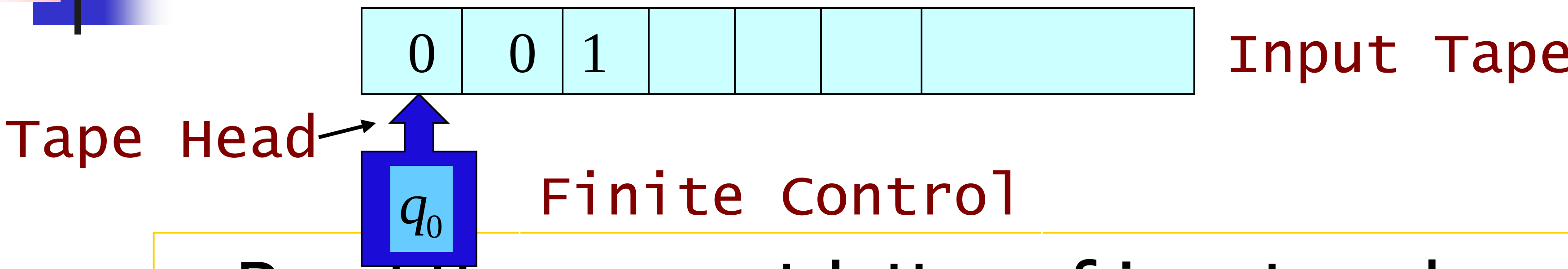
Finite Automata

Formal Specification of Languages

- Generators
  - Grammars
    - Context-free
    - Regular
  - Regular Expressions
- Recognizers
  - Parsers, Push-down Automata
    - Context Free Grammar
  - Finite State Automata
    - Regular Grammar
- A Finite Automata is:
  - a mechanism to recognize a set of valid inputs before carrying out an action.
  - a notation for describing a family of language recognition algorithms.

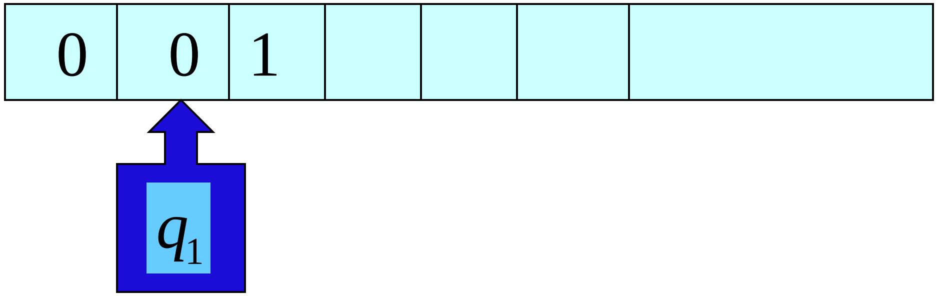
Operation of the machine



- Read the current letter of input under the tape head.
- Transit to a new state depending on the current input and the current state, as dictated by the transition function.
- Halt after consuming the entire input.

Operation of the machine

- Transitions show the initial state, input, and next state
  - Form: $\delta(q,a)=b$
- Example:
  - $\delta(q_0,0)=q_1$ $\delta(q_0,1)=q_2$
- Tape head advances to next cell, in state q_1
- What happens now?
  - What is $\delta(q_1,0)$?



Associating Language with the DFA

- Machine configuration:

$$\boxed{[q,\omega]} \text{ where } q \in Q, \omega \in \Sigma^*$$

- Yields relation:

$$[q, a\omega] \mapsto^*_{\mathrm{M}} [\delta(q,a), \omega]$$

- Language:

$$\{\omega \in \Sigma^* \mid \underbrace{[q_0, \omega] \mapsto^*_{\mathrm{M}} [q, \lambda]} \wedge q \in F\}$$

Deterministic Finite Automaton (DFA)

$$\boxed{M = (Q, \Sigma, \delta, q_0, F)}$$

Q:Finite set of states

Σ: Finite Alphabet

δ: Transition function

- a total function from $Q \times \Sigma$ to Q

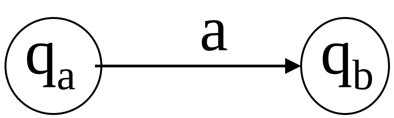
q_0 :Initial/Start State

F :Set of final/accepting state

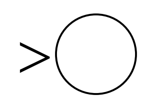
Finite State Diagram

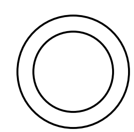
- A graphic representation of a finite automaton

- A **finite state diagram** is a directed graph, where nodes represent elements in Q (i.e., states) and arrows are characters in Σ such that:



Indicates: $((q_a,a),q_b)$ is a transition in δ

The initial state is marked with: 

The final state(s) are marked with: 

Automata

- Deterministic automata – each move is uniquely determined by the current configuration
  - Single path
- Nondeterministic automata – multiple moves possible
  - Can’t determine next move accurately
  - May have multiple next moves or paths

Automata

- An automaton whose output response is limited to yes or no is an acceptor
  - Accepts input string and either accepts or rejects it
- Measures of complexity
  - Running time
  - Amount of memory used

Automata

- Finite automaton
  - Uses a limited, constant amount of memory
  - Easy to model
  - Limited application

Automata

Given an automaton $A = (Q,\Sigma,\delta,q_0,F)$, and a string $w \in \Sigma^*$:

- w is **accepted** by A if the configuration (q_0,w) yields the configuration $(F, \in)$, where F is an accepting state

- the **language accepted by** A, written L(A), is defined by:

$$L(A) = \{w \in \Sigma^* \mathbf{:} \text{ w is accepted by A}\}$$

Automata

- Deterministic finite automaton
  - Every move is completely determined by the input and its current state
  - Finite control device
    - Can be in any one of the states, $q \in Q$
  - May contain trap or dead states
  - Contains accepting state(s)

Examples

Deterministic

Language Acceptor

Design a FSA to accept strings of a language

- strings of a's and b's that start and end with a

$$M = (Q, \Sigma, \delta, q_0, F)$$

*Q: S*tates are required for the following:

- q_0: Start state
- q_t: Trap for strings that start with a b
  - Accepting state can't be reached
    - Machine only accepts strings that start with an a
- q_f: State reached after any a in a string that started with an a
  - The final state
- q_1: State reached after any b in a string that started with an a

Language Acceptor

Σ : {a,b}

δ : $\delta(q_0, a) = q_f$ Is the string *a* in the language?

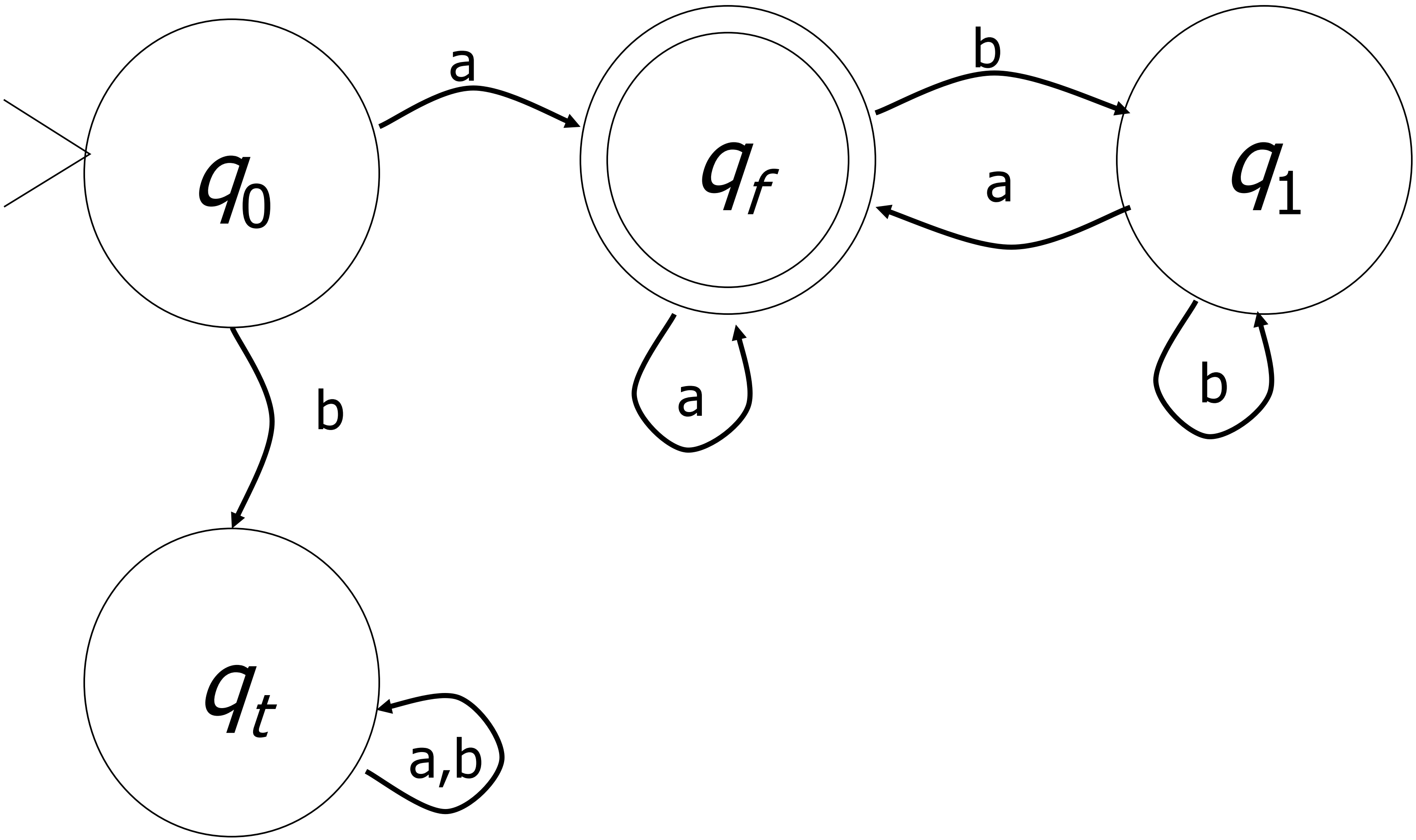
$\delta(q_0, b) = q_t$

$\delta(q_t, a) = q_t$ $\delta(q_t, b) = q_t$

$\delta(q_f, a) = q_f$ $\delta(q_f, b) = q_1$

$\delta(q_1, a) = q_f$ $\delta(q_1, b) = q_1$

Language Acceptor



Vending Machine

- Suppose:
  - All items cost 40¢
  - Coins accepted are 5¢, 10¢, 25¢
  - Recall $M = (Q, \Sigma, \delta, q_0, F)$
  - What are these entities?
  - Q is a set of states
    - What are the possible states?
  - Σ is the alphabet
    - What are the input symbols?
  - δ are the transitions
    - How do we move from state to state?
  - q_0 is the starting state
    - Where does the machine start from?
  - F is the final state
    - When does the machine stop?

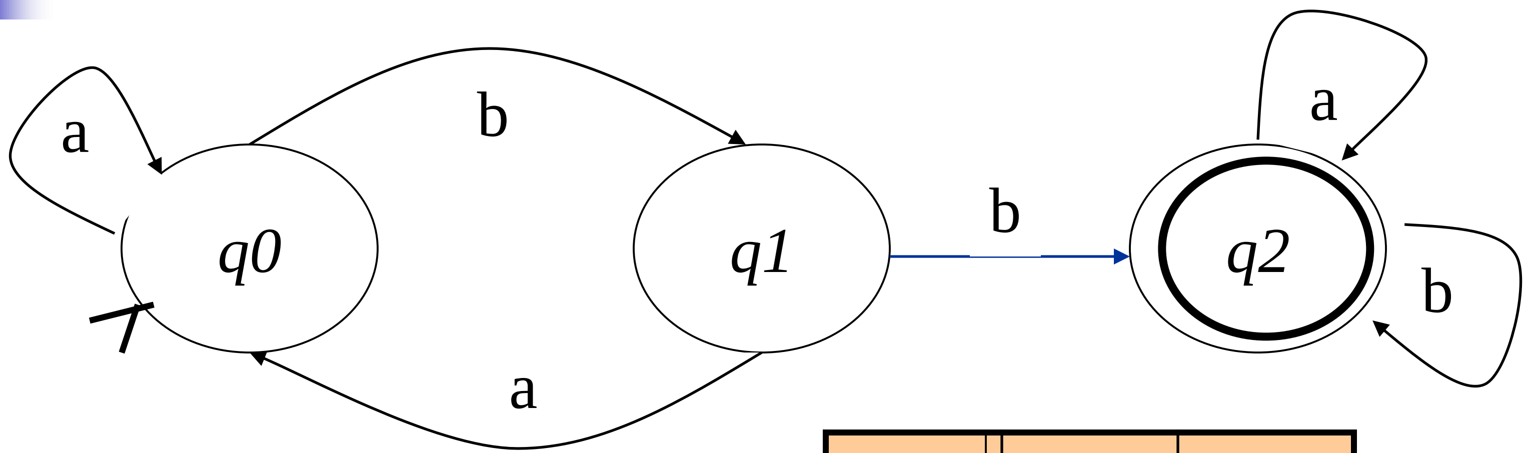
Vending Machine

- Q: What are the possible states?
  - The status of the machine before and after any of the alphabet symbols have been applied
    - The present state represents how much money has been deposited
    - Could also represent how much is left to deposit
- Σ: What are the input symbols?
  - The coin denominations
- δ: How do we move from state to state?
  - Transition when a coin is deposited
- q_{0}: Where does the machine start from?
  - The beginning!
- F is the final state
  - When does the machine stop?
    - Not before you've deposited enough money
- Wait! What if you put in more than 40¢?

Set of strings over {*a*,*b*} that contain *bb*

- Design states by parititioning Σ^*.
  - Strings containing *bb* *q2*
  - Strings not containing *bb*
    - Strings that end in *b* *q1*
    - Strings that do not end in *b* *q0*
  - Initial state: *q0*
  - Final state: *q2*

State Diagram and Table



$$Q = \{q0, q1, q2\}$$

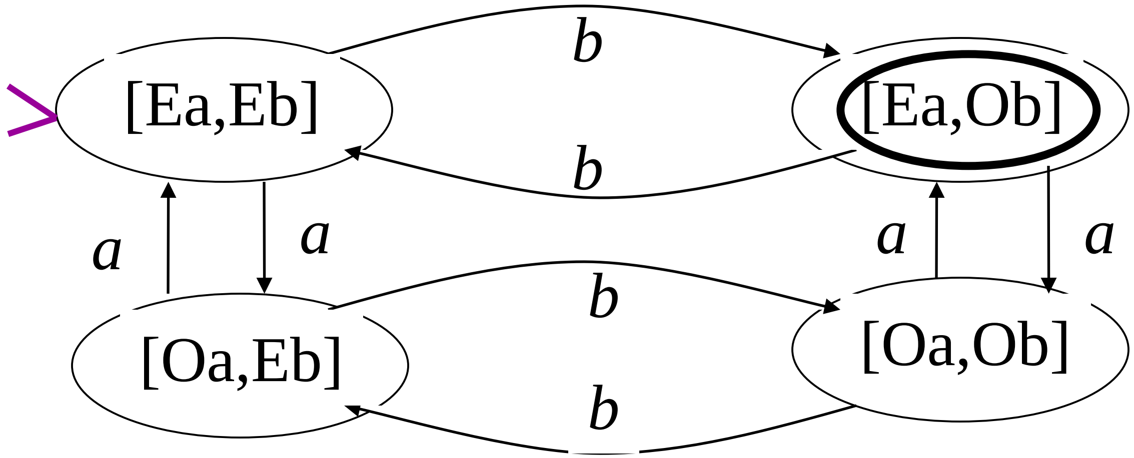
$$\Sigma = \{a, b\}$$

$$F = \{q2\}$$

$$[q0, aab] \mapsto^* [q1, \lambda]$$

δ	a	b
q0	*q0*	*q1*
q1	*q0*	*q2*
q2	*q2*	*q2*

Strings over {*a*,*b*} containing even number of *a*'s and odd number of *b*'s.



Non-Determinism

Automata

- Non-deterministic finite automaton
  - More than one destination from a state with a distinct input
  - At least one state has transitions that cannot be completely determined by the input and its current state
  - It is possible to design a machine where a single input can have two paths to an accepting state
  - $\in$ transitions
    - Move from a state without input

NDFA

- Nondeterministic finite automata
  - Quintuple $A = (Q, \Sigma, \Delta, s, F)$ where

    Q is a finite set of states

    Σ Is an input alphabet

    $\Delta \subseteq Q \times (\Sigma \cup \{\in\}) \times Q$ is the transition relation

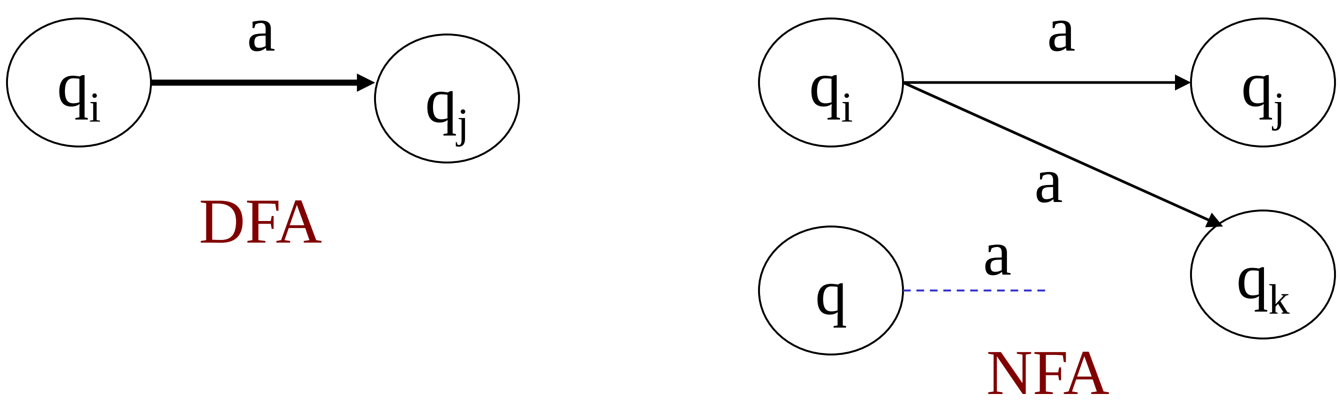
    S $\in$ Q is the initial state of the automaton

    $F \subseteq Q$ is the set of favorable states

NDFA

- The range of Δ is not a single element of q, but a subset of q
  - Example: Δ (q1,a) = {q0,q2}
    - Either q0 or q2 could be resultant state
- Empty string (null input) can result in a change of state

NFA



$\delta_{DFA} : Q \times \Sigma \rightarrow Q$ total function

$\delta_{NFA} : Q \times \Sigma \rightarrow Pow(Q)$ total function

$\delta_{NFA} \subseteq Q \times \Sigma \times Q$ subset relation

NDFA

- Stuck state – no jumps out of state labeled with the input symbol
  - Can not lead to accepting state
  - Removes need for trap state
- Input is accepted if any resultant configurations lead to an accepting state

NDFA

- Given an NFA $A = (Q,\Sigma,\Delta,q_0,F)$, and a string $w \in \Sigma^*$:
  - w is **accepted** by A if at least one of the configurations yielded by (q_0,w) is a configuration of the form (F,ε) with f a favorable state
  - $L(A) = \{w \in \Sigma^*$ **:** w is accepted by A$\}$

Examples

Non-Deterministic

Language Acceptor (Revisited)

Design a FSA to accept strings of a language

- strings of a’s and b’s that start and end with a

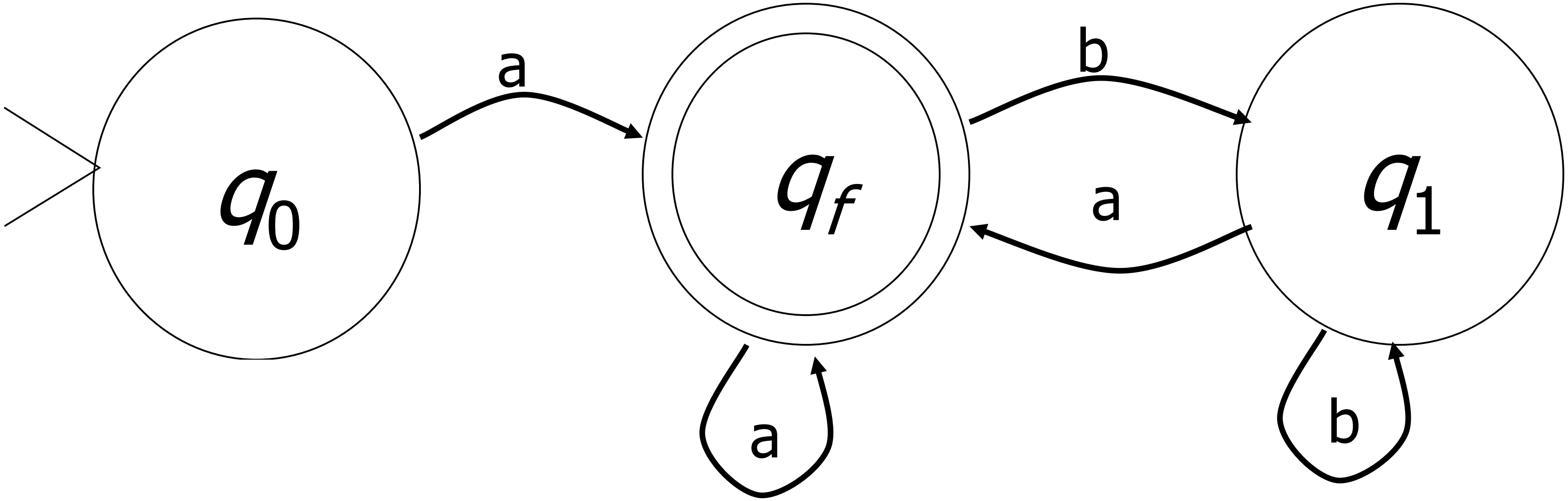
$$M = (Q, \Sigma, \delta, q_0, F)$$

- Only change is in δ morphing to Δ

$$M = (Q, \Sigma, \Delta, q_0, F)$$

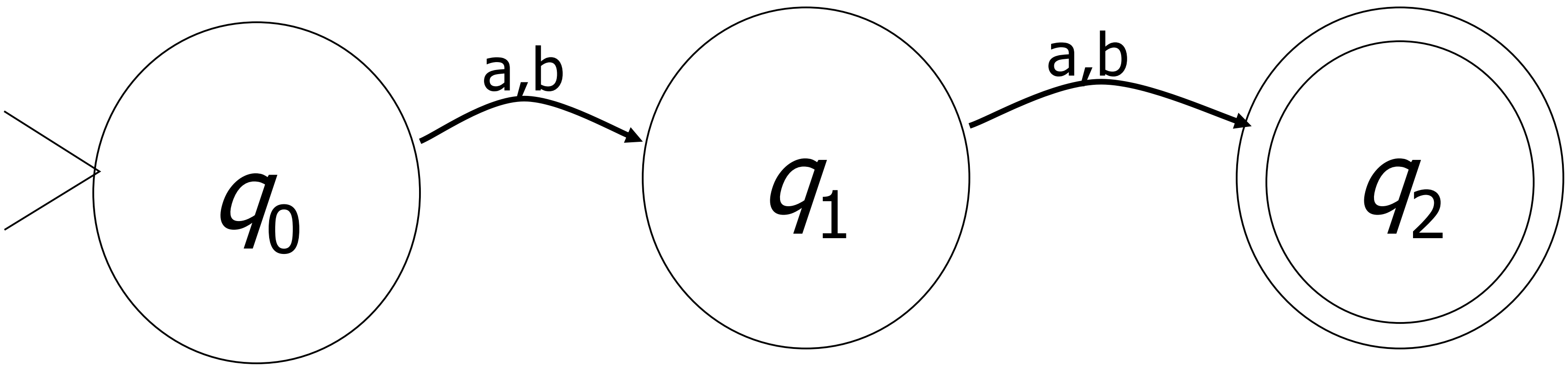
- Some transitions eliminated

Language Acceptor



Another Language Acceptor

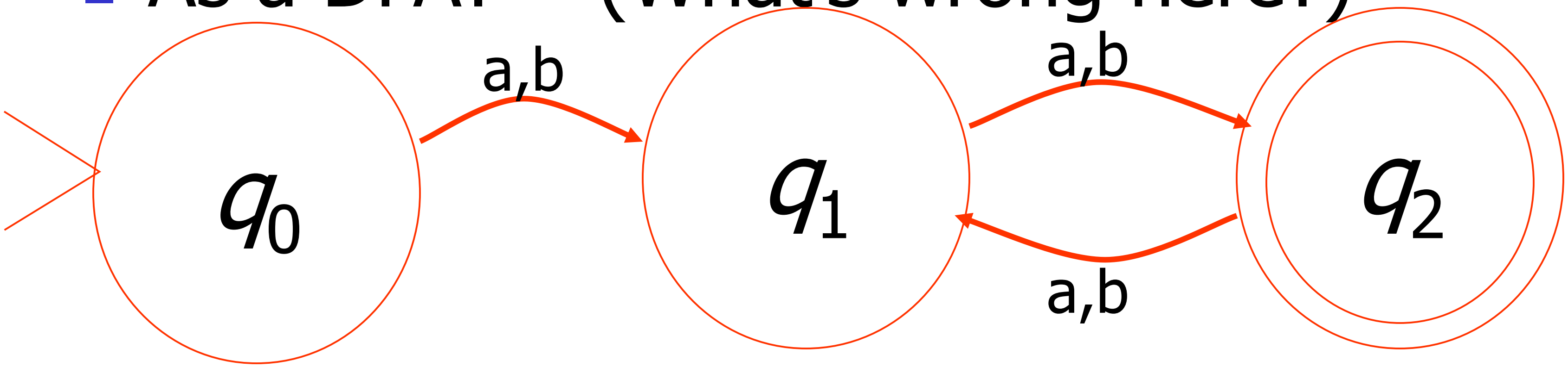
- Build a FA to accept strings of even length



- Wait! This only accepts strings of length 2
- How to update?

Another Language Acceptor

- As a DFA? (What's wrong here?)



- As an NFA (use ∈ transitions)